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ON SENSIBILITY[1]

"MIND" REQUIRED TO EXPRESS DIFFERENCE OF LIFE

ONE who has tried to define life, with all its purposefulness and freedom, as a thing residing in and moving through mechanism without ceasing to be part of it, can not be unacquainted with trouble. Yet, this task accomplished as best it might be, he would be innocent indeed who did not know that if he would go on to define mind in the same empirical way the worst of his perplexities lay before him. For to define a thing in an empirical or pragmatical way is so to define it as to leave no doubt what experiments would inform us whether anything corresponding to our definition existed or no. But where minds are concerned we have not yet or only recently grown scrupulous to avoid making hypotheses about them that suggest nothing in the way of an experiment by which our guesses might be tested. We have pretended to know too much of our own, been content to know too little or nothing at all of our fellow's mind, and empirical health is not in us. So that one to whom his own mind is no immediate certainty and his neighbor's no unapproachable eject has trouble in explaining how and why both of them are objects to be experimented upon like trees or houses—and yet very unlike, too, in the kind of experiment they call for.

As these matters are not old and well-worn, but (if I may judge from the criticism previous papers of my own and of other sympathetic writers have called forth) new and even repugnant to prevailing habits of thought, there may be some advantage in beginning the discussion of them where otherwise one might rather have hoped to leave off. A tentative generalization at the outset has often helped me, a reader, and I see no reason why the like should not be vouchsafed by me, a writer, if only it will not be taken with too much finality. But if I start with some very loose reflections on the nature of mind, it will only be to abandon them as quickly as may be and

[1] Paper written for the Philosophical Club of Columbia University. It presupposes the arguments of two earlier ones: "Mind as an Observable Object," this JOURNAL, Vol. VIII., p. 322, and "The Pulse of Life," this JOURNAL, Vol. XI., p. 645.

to devote the body of my paper to one of the most special and technical problems with which an empirical method can be confronted.

It will be remembered that the empirical method of defining mind begins by identifying what you would do to find out whether a being had a mind or not with what you mean by mind, or, as I have sometimes put it, "the criterion of mind constitutes its definition." Now those who have approached the problem of the criterion of mentality fall into two classes: some think they find mind wherever they find life and others attribute mind to only those forms of life they call the higher. Those who hold the first opinion seem to mean not only that life and mind have the same denotation, but also that there is no difference in the connotation of the two terms. If so, the fathers who invented our categories for us were for once lavish in giving us two names for the same thing, whereas it was their niggardly custom to make out with one word for ten thousand things. On the other hand, those who attribute mind to the higher forms of life and not the lower escape the danger of voiding the distinction between life and mind only to fall into another more serious. For I know of none of them whose test of the presence of mind remains clear without becoming structural. But if there is anything in the poet's saying that tired limbs were invented before feather beds and that thirst is older than wroughten cups, it may be our fathers knew a great deal about minds before they knew anything of delicate anatomy.

There is a way, however, of doing justice to both of these historic motives; the denotation of mind may be the same as that of life, and yet the connotation remain quite different. For if life were all of a level, if all things pursued the same end with no difference of skill, I can not see how we should ever have come by the concept of mind or in what way it would have served us if we had. But in our world there are purposeful beings that differ in their aims and there are beings pursuing the same end that differ in the skill with which they work for it. Take just the latter of these classes (for I must postpone the discussion of the former to another occasion); there is no mistaking the need the biologists have felt to account for the different resourcefulness of beings struggling for the common end of self-preservation by introducing the category of mind. Is it not because this being has a mind the other lacks that it can adapt itself and live where the other fails and dies?

It is the observation of a certain difference of lives that has stung the primitive biologist into inventing mind. But, alas, if we ask his modern representative for a definition of this new aspect of life he either falls into mysticism or returns an answer that amounts to saying: Mind is that to have which is to have what accounts for a greater resourcefulness of behavior. Having arrived so far, he feels

that any further pursuit of a problem so lacking in empirical suggestiveness is beneath his dignity as a scientist and he is ready to turn the whole matter over to a certain entity he calls the "metaphysician."

But what if, having learned a lesson from history, we were to reverse all this; what if instead of saying, "this being is more resourceful than that because it has a mind the other lacks," we were to say, "this being has a mind the other lacks because it is more resourceful." Then all we should mean by the various faculties and graded refinements of mind would be the empirical description and measure of these differences in the conduct of life. To be sure, this would mean that if there were no comparison of minds there would be no mind at all, that we should have no reason for attributing mind to anything if we did not give it more or less than some other. But though it would be absurd to say the like of one's keys or purse or medulla, there are many concepts beside that of mind that owe their meaning to relations. Thus many of us after reading Berkeley are content to say that nothing would have any reality at all had it not more or less than some other, and would feel that we had by honest effort struggled out of the metaphysical mood that would express reality in terms of a Lockian substance whose only definition was to be that which makes things real.

To come, then, to a definition (or rather partial definition) of mind. If one being can accomplish a given purpose in $(n+1)$ types of situation, another in but n of these, we shall call that in which the first is better equipped than the second a faculty of mind. I call this definition partial because (1) it makes comparison of means only, not of ends, and (2) it presupposes two minds, the observing and the observed whose relationship is not explained.[2] But I foresee nothing in the later discussion of these matters that will contradict what has so far been said, although if I knew how to say everything at once I should probably want to put each thing a little differently. Lacking this gift, I do not see how any scientific discussion can avoid a certain "dialectic" that asserts only to take back in part.

Intensity of Sensation Defined

If I turn back to very ancient history to illustrate the idea set forth, I fall first upon Aristotle's distinction between animals and vegetables. With Aristotle organisms were generically alike in purpose and specifically different in resource. The first gain of resourcefulness he noted he set down to the faculties of sensibility and spontaneous motion that raised the animal above the vegetable species in

[2] *Cf.* however, this JOURNAL, Vol. X., pp. 141 ff.

the scale of life. Our modern love of continuity does not encourage us to look upon the acquisition of faculties as so sudden an affair, but we do still regard differences in the delicacy and variety of sensibility as important to the accomplishment of purpose and as evidence of mental superiority. So, too, spontaneity, originality, inventiveness are made measures of intelligence and are taken to be faculties that make for success. This pair, sensibility and spontaneity, might profitably be driven abreast, for I doubt if we find either of them very far apart from the other; but as I must limit myself to one (not to lengthen the discussion) I choose sensibility. Nor do I conceive that I am proposing any different problem if I offer to discuss what it means to have a sensation of a certain quality and a certain intensity.

When the sun rose on Austerlitz, it warmed a puddle and set up chemical reactions in its shallow depths; infusoria stirred; an emperor made a gesture. Inanimate matter "reacted" physically and chemically; animalcules after the manner of their various tropisms; the emperor in the fashion of a Napoleon. Why do we say that of these various reactions, that of the inanimate stuff shows something less than sensibility, that of the Emperor waving on his armies very much more, while just the simplest life there was there shows pure sensibility, nothing more and nothing less?

The answers may well lead up to a definition of sensibility. And I should say: (1) We do not call the physical and chemical reactions sensitive because they are not teleological. (2) We do not call the Emperor's gesture mere sensibility because, though full of purpose, its stimulus was not the sun's light or heat, but the sun and a fair share of the universe. (3) But we do call the behavior of the simple organism a display of sensibility because, viewed as a reaction, it was teleological, and considered as to its stimulus, this stimulus was confined to mechanical changes at and within the surface of its body. The proof of which is that if we imagined all the rest of the cosmos changed, but in such wise as to leave the mechanical situation at the organism's surface unchanged, we should expect the organism to behave in the same way. It has a sensation, not a perception.

Whence this definition of sensibility: Any body that reacts with a purpose we call its own to a change of mechanical conditions within its contours displays sensibility, or has a sensation.

It follows from this that all living beings are sensitive or have sensations, but it follows equally from what went before that we should have no reason for saying so did not different living beings display such differences of sensibility as lead us to recognize sensations of varying quality and of distinguishable intensity. Wherefore it is to the definition of quality and intensity of sensation that our argument would carry us on.

The older empiricism of a Locke or of a Condillac did everything it could to foster the idea that experience began with sensations. And so it does, very possibly; but it takes a most experienced person to tell what is meant by the "simple idea" with which experience begins. What is the quality, what is the intensity of this first sensation? The learning of the ages is little enough of an equipment for the telling. It is only one who knows a great deal about the world "made out of" sensations that knows anything about the "sensations" this world is made out of. Wherefore one can not avoid, nor should any one want to avoid, bringing as much as he can of his knowledge of mathematics and physics to bear on the task of telling what it is that a human baby or maybe a well-grown ameba has when it is said to have a sensation of this or that quality and of greater or less intensity.

To begin with intensity: only one who has developed the concept of intensity in the physical order can tell what it means in the psychical. Now to explain why, if I increase the amplitude of a light wave, I add to its intensity, while, if I alter its wave-length, I change its quality, is a matter that calls for some consideration. The old opposition between quality and quantity can not have any application to a state of affairs in which every term is quantitative. Why does the amplitude series constitute a scale of intensities, differences of wave-length a spectrum of qualities?

If I am not mistaken, the best English into which we can render the Latin *quale* is not *what*, but what *not!* It would seem that we endeavor to throw what differences we can into various scales of intensity and what we can not we leave qualitative, *i. e.*, *not* intensive. Now a scale of intensities has this that is peculiar to it; its different members are distinguished in terms of energy. But where energy differences are not apparent in a physical series, we allow it to remain a scale of qualities. In this way we can understand how it is that two light waves of different amplitude, but constant wave-length, should be regarded as of different intensity, while two light waves of different period, but the same amplitude, should be called qualitatively different.

Having thus defined intensity of stimulus, we are free to inquire what it is in an organism's reaction to stimuli of varying intensity, or to stimuli of the same intensity under varying conditions of sensibility, that leads us to attribute to this organism sensations of different intensity.

The classic model of such an inquiry is Fechner's, the outcome of which was to define the intensity of sensation produced in a given organism by a stimulus of a given intensity in terms of the following empirical data: (1) The intensity of the physical stimulus measured

in a certain physical scale; (2) the intensity of the stimulus just noticeably greater than this; (3) the intensity of the stimulus just noticeable by the organism under the same conditions of sensibility. This definition of Fechner's meets, I conceive, all the formal demands that the traditional meaning of the term "intensity of sensation" imposes, and we need have undertaken no further labors in this field were it not for one unfortunate thing. And that is that the third of our empirical data can not in general be obtained. It could only be obtained for all possible cases if Weber's law could be established for all ratios of the j. n. g. stimulus to the standard, from the lower threshold stimulus to the upper. This is apparently more than can be done by experimental means so far devised, whereupon one may sum up by saying, Fechner has defined something that if it existed would be a sensation of measurable intensity; but no such thing exists.

Thus the empirical method requires us to undertake anew a reduction of vague traditional ideas respecting intensity of sensation to an exact definition that will satisfy them as well as Fechner's did, and at the same time, by avoiding all reference to Weber's law, permit us to find in experience objects corresponding to the thing we have defined. This I have tried to do with results that I should like to submit to your judgment; I found the task difficult indeed, but so fascinating that it is only by the exercise of a self-restraint I hope will be counted unto me for righteousness that I abstain from plunging into a full discussion of the way this defining formula was arrived at. However, I have never found that philosophers in general shared my fondness for getting my thoughts as quickly as possible into mathematical form and keeping them there as long as they would let me. Therefore, I have contented myself with submitting along with this paper the discussion referred to in case any would do me the favor to look at it: it will be enough for the present purpose if I set down the formula quite baldly, only lingering lovingly on some of its more obvious charms and virtues.

$$I_r = \frac{1}{\log \dfrac{r_+ - r_0}{r - r_0}} \log \frac{r - r_0}{r_\omega - r_0}.$$

That is, the intensity of the sensation due to a stimulus r is a function of exactly the same form as Fechner's and would be identical with his, if r_0 were the zero of the physical scale with respect to which Weber's law was supposed to hold and r_ω were the "threshold stimulus." I have tried to show that any definition of intensity of sensation must have this form; but that we may find a new interpretation for r_ω and r_0 that (1) makes no reference to Weber's law, (2) is indefinitely more liable to be found applicable to experience, and

(3) enables us to determine the value of I_r by observing the reactions of the organism to variations of the stimulus through an indefinitely small range of values.

The data that must be collected experimentally in order that the intensity of a given sensation may be calculated can be illustrated in a particularly simple case. Jennings has described the behavior of the paramecium under the influence of temperature stimuli. Placed in a trough of water kept at a temperature within a certain range, the paramecia will be found evenly distributed throughout the trough. But if one end of the trough be heated and the other cooled beyond the limits of this range, the paramecia will be found gradually to collect in the region of optimum temperature. This phenomenon comes as near as can be to an exhibition of pure sensibility; in the change of temperature of the medium we have a stimulus describable in terms of mechanism; in the motion of the organism, a reaction obviously teleological, and finally, in explaining this reaction we need take into account nothing in the way of stimulus but what happens at the surface of the organism itself.

It is possible then to ask, what is the intensity of the sensation produced in a paramecium by a temperature lying within a certain range? And to this question it is incumbent on us to find an experimental answer. But this is exactly what our definition of intensity enables us to do very simply. For, suppose the organisms evenly distributed at a given temperature; suppose, then, the temperature increased at one end of the trough until the tendency is noticed for motion to take on a common direction, when we may say the organism has "just noticed" the change in its environment. The other data required for our calculation would be gathered by again decreasing the temperature until the original distribution is recovered at a temperature which, if our formula is applicable, will not be identical with that from which we set out, but which we may define as producing a sensation of the *same intensity* under conditions of "over-estimation." Again increasing the temperature under these new conditions until the change is "just noticed," we have one group of data, and repeating the whole experiment for a range of temperatures as narrow as we please we have all the data required for our calculation. Our definition still stands a chance of proving inapplicable, though faint indeed as compared with Fechner's. Should it prove inapplicable we should have to find new interpretations for some of the r's of our formula; but if my analysis is correct, we should be confined to a definition of the same form.

I have chosen this very simple example of what an empiricist would mean by attributing a sensation of a certain intensity to a given living being because the behavior on which his attribution rests is here so obvious. But the case is not so simple as to be free from all suggestion of difficulty. Indeed it can be made to yield one question that seems to me of great importance in defining the sort of behavior on which an empiricist bases his judgment that he is dealing with a phenomenon of mind.

"For suppose," a critic may well ask, "suppose you had not carried out the series of experiments on which you based your calculation of the paramecium's sensation, suppose you had not varied the temperature of the medium—then there would have been no behavior on the part of the paramecium for you to observe. Would the organism have been without a sensation of a certain intensity just because you had taken no measures to ascertain what it was? On the other hand if you say it had the sensation when it did not exhibit the behavior, how can you identify sensation with behavior?"

Here, if I am not mistaken, we come upon the reason for regarding sensibility as "passive," having a sensation as a receiving rather than a doing of aught in the world. For the behavior, I would answer, that measures the intensity of our organism's sensation is not necessarily *its* behavior at all, but the behavior of certain classes of beings to which it belongs.

I have already suggested that all studies of teleologically defined beings are statistical.[3] It will surprise no one then if I maintain that the order of facts on which depends my calculus of a paramecium's mental condition is quite analogous to that on which I base my estimate of a man's chance of life at a given moment. This chance of life is just as much the man's private possession as is his purse or his keys or his medulla; but whereas I can determine these latter properties without placing him in various classes to be studied by the method of averages, I can not mean anything by his chance of life save such a calculus of probabilities as actuarial experience justifies. Just so, the greater part of the "behavior" referred to by the empiricists who would define mind in terms of it, is not *actual* behavior on the part of the being to whom this mind belongs, but *virtual* behavior, which means no more than that it is the behavior we should have reason to expect if such and such experiments were tried. But what I have called our reason to expect certain reactions can be but the experience of what actually has happened in other cases: it is a calculus of probabilities based on statistics and may be made as accurate a calculus based on as rich a collection of statistics as our

[3] This JOURNAL, Vol. XI., pp. 645 ff.

practical interest in the case justifies. It is only an old confusion of categories that can make it seem odd that while it is *A*'s mind we are speaking of, it is not by examining *A* alone, but by studying *B*, *C*, and *D*, that the answer to our question about *A* can be obtained—and that not an approximate or makeshift answer; but the only kind of answer that makes that kind of question meaningful.

Quality of Sensation Defined

While I do not propose to let myself off with a discussion of sensibility in its simplest forms only, perhaps it will be better to say what little it is necessary to say respecting the quality of sensation before departing from the domain of the simple. Still clinging, then, to my paramecium, it will be observed that while I was willing to discuss the intensity of its sensation due to a temperature stimulus, I was careful to avoid speaking of this as a sensation of heat. For imagine an organism so simply constituted that although it reacts to stimuli of different quality (light, heat, chemical, electrical) it responds to all of them in the same way. Let us suppose its only way of reacting to any stimulus is to move, would there be any reason for attributing to it different qualities of sensation because it moved in response to different qualities of stimulus? If we had to find a name for the quality of its sensation, would it not be enough to call it a sensation of discomfort whether light, heat, chemical changes, electricity, or whatever else was responsible for its state?

It is only, then, when a subject responds to one quality of stimulus in a way it does not react to another that we have any reason for attributing to it sensations of different quality. As the development of reactions specific to different qualities of stimulus goes hand in hand with the development of specific sense organs we are only too likely to fall back on a structural definition of the qualitative differences of sensation. The sensation that results from stimulating the eye has the quality of color, while what comes from stimulating the ear has the quality of sound. *A la bonne heure;* but a man who reacts in the same way to a stimulus of red and to one of green is easily recognized to be color-blind long before we know anything about the structural defect in his retina that goes with this infirmity. In short, it is the difference in reaction to different qualities of stimulus that defines qualitative differences of sensation: what relation this difference bears to structure is a secondary question.

Finally, it is evident from our definition of quality as the what *not*, increase in the variety and richness of experience is constantly changing the meaning of each quality as it is differentiated from more and more others. Whence nothing could be more misleading than the account our older empiricism offers of the way in which ex-

perience beginning with simple ideas (or as Helmholtz called them *qualia*) leaves these unmodified and only learns to recognize new combinations of them. It can be no truer that I begin with *qualia* and come by a knowledge of the world, than that I begin with a world and come by a knowledge of the *qualia*.

The Higher Forms of Sensibility

I have, I see, allowed myself but scant space in which to discuss the phenomena of sensibility in the higher forms of life; but fortunately little more is needed than to adapt the principles already laid down to somewhat changed conditions. As long as I confined myself to those very simple forms of life, the exciting cause of whose behavior lay in their immediate surroundings and was susceptible of a purely mechanical description, it was not hard to tell what behavior I meant when I defined their conscious life in terms of their actual and virtual behavior. But as we ascend in the scale of life the stimulus to actual behavior becomes more and more complex and more and more of the mechanical conditions at the surface of the subject's body may change without altering the trend of his present conduct; while a larger and larger share of the universe remote from the subject can not be conceived different and his conduct remain the same. Thus as I sit writing, the blue of the sky out of the window, the crackling of the logs on the hearth, are of so little importance to what I have on hand that the sky might lose much of its light and the fire die down to silence and yet no change in my behavior be noted. Yet it is not denied that I do receive sensations of some sort from these and a thousand other things in and around me, so that the impression is deepened that to have a sensation is a very passive sort of experience.

Nor, being rather a lover of old terms than an innovator, do I see why sensations should not even from the point of view of the kind of empiricism I have been arguing for continue to be spoken of as passively received impressions of a certain quality and of a certain intensity. But I should interpret this to mean that the behavior one appeals to to define mind in its various aspects must of course include the behavior that doesn't take place as well as the behavior that does. What indeed is meant by asserting that the sky's blue is less intense and qualitatively vaguer to me than to my artist neighbor who is trying to paint it? Is it not that a change in this intensity or in this quality which would make him reach for a new color would leave me scribbling with the same pen? But some degree of change would disturb me too; if there were no way of estimating this virtual behavior of mine (as there would not be if I were blind) I should not be credited with a sensation of light. After all, sensibility in man

is very much like sensibility in the paramecium and I hardly regret that I have left myself so little space in which to deal with it.

The Inner Mental Life

Having come so far on the way to a definition of sensation, its quality and intensity, I was sensible the way had been dry and if I was not to overtax the patience of even such tried travelers as those who had invited me to lead them for once, it was time I hastened on to some sort of a stopping place. Therefore with a brief reference to the sensory material of dream life as presenting a most obvious case in which neither the stimulus nor the reaction in terms of which I had defined the quality and intensity of sensation was open to inspection, I left my argument to the judgment of those for whom half a word is more than enough. But as it has seemed to very distinguished writers (such as Henri Bergson) that the recognition of sensation as existing where neither stimulus nor reaction is observable to the onlooker was an altogether convincing argument against going to work on the definition of sensation in the way I have, I ought, I think, to take a more leisurely survey of this field and to devote a separate article to a consideration of the inner mental life.

Supplement

Definition of Intensity of Sensation

If r is a stimulus measured in some physical scale of intensity, and if $f(r_1, r_2)$ is the difference of intensity produced by the stimuli r_1 and r_2, respectively, then

Post. 1.—$f(r_1, r_2)+f(r_2, r_3)=f(r_1, r_3)$

which has for its solution

$$f(r_1, r_2)=\phi(r_1)-\phi(r_2).$$

If $\phi(r)$ is to have the properties traditionally given to the intensity of a sensation produced by the stimulus r, then

Post. 2.—The intensity of the sensation produced by stimulus r is independent of the *zero* of the physical scale in which r is measured. Wherefore

$$\phi(r)=\phi(r_\alpha - r_\beta).$$

Post. 3.—The intensity of the sensation produced by stimulus r is independent of the *unit* of the physical scale in which r is measured. Wherefore

$$\varphi(r) = \varphi\left(\frac{r_\alpha - r_\beta}{r_\gamma - r_\delta}\right).$$

To interpret r_α, r_β, *etc.*, empirically, let the place of r_α be taken by the stimuli r_1, r_2, *etc.*, and let r_{1+}, r_{2+}, *etc.*, be their just noticeably greater stimuli. Let $r_\beta = r_\delta = r_0$, in which r_0 is some zero of the physical scale in which r is measured, such that the equation holds

$$\frac{r_{1+} - r_0}{r_1 - r_0} = \frac{r_{2+} - r_0}{r_2 - r_0}.$$

If Weber's law held and if we imagined r_1 to remain constant while r_2 varied, r_0 would have a constant value expressed by the equation

$$r_0 = \frac{r_1 r_{2+} - r_{1+} r_2}{r_1 + r_{2+} - r_{1+} - r_2}.$$

But it is less of an assumption to suppose, not that r_0 remains constant under these conditions, but that it approaches a limit as r_2 approaches r_1. In place of Weber's law we take, then,

Post. 4.—

$$r_0 = \lim_{r_2 \doteq r_1} \left(\frac{r_1 r_{2+} - r_{1+} r_2}{r_1 + r_{2+} - r_{1+} - r_2} \right),$$

and substituting the *r's* so far defined, we have

$$\varphi(r) = \varphi\left(\frac{r - r_0}{r_\gamma - r_0} \right).$$

Interpreting r_γ to be the intensity of the stimulus that produces a sensation of zero intensity and writing $r_\gamma = r_\omega$, we may set down

Post. 5.—

$$\varphi\left(\frac{r_\omega - r_0}{r_\omega - r_0} \right) = 0,$$

and substituting r_ω for r_γ, we have

$$\varphi(r) = \varphi\left(\frac{r - r_0}{r_\omega - r_0} \right).$$

Post. 6.—The intensity of the sensation produced by a changing stimulus is a continuous function of that stimulus.

Therefore, since our data are experimental with a probable error attached we may always find an analytic function of r lying within the limits of p. e. that will satisfy the conditions imposed upon $\phi(r)$.

Post. 7.—Subjective conditions (of sensibility) remaining constant, the intensity of sensation increases with the stimulus.

Post. 8.—Subjective conditions remaining constant the *rate* at which the intensity of sensation increases with the stimulus is independent of what we take to be the sensation of zero intensity. Since

r_ω is defined as the stimulus producing a sensation of zero intensity, Post. 8 amounts to saying

$$\frac{\partial}{\partial r_\omega}\left(\frac{\partial}{\partial r}\varphi\left(\frac{r-r_0}{r_\omega-r_0}\right)\right)=0.$$

From the postulates so far laid down, we have for $\phi(r)$ the unique solution

$$\varphi(r)=a\log\frac{r-r_0}{r_\omega-r_0}.$$

But our formula has as yet failed to make explicit those "subjective conditions" of sensibility that have been traditionally accepted as playing a part in the intensity of sensation produced by a given stimulus. As these have been made to depend on the j. n. g. stimulus r_+, we must determine how this r_+ enters into our expression. To do this consider the case in which a unit difference of intensity exists between $\phi(r_2)$ and $\phi(r_1)$, $(r_2>r_1)$, subjective conditions remaining constant. That is,

$$a\log\frac{r_2-r_0}{r_\omega-r_0}-a\log\frac{r_1-r_0}{r_\omega-r_0}=1,$$

in which case

$$a=\frac{1}{\log\dfrac{r_2-r_0}{r_1-r_0}}.$$

Here the only arbitrary quantity is r_2, so that if the j. n. g. stimulus is to appear in our formula at all it must be included in our definition of r_2. Whence

Post. 9.—$r_2=f(r_{1+})$.

But since from Post. 2, the r's of our formula can only enter in the form r_a-r_ω, it follows that

$$f(r_{1+})=r_{1+}$$

and if we now designate by the symbol I_r, the intensity of the sensation produced by stimulus r, we have the definition of I_r in the form

$$I_r=\frac{1}{\log\dfrac{r_+-r_0}{r-r_0}}\cdot\log\frac{r-r_0}{r_\omega-r_0}.$$

There only remains to be selected the empirical meaning of r_ω, Fechner took r_ω to be the threshold stimulus and so made the applicability of his definition to hang on the truth of Weber's law. A phenomenon much better established by experience is that of the

over-estimation of the second stimulus. Let r' be the stimulus that under conditions of over-estimation is judged equal to r; r_+' the stimulus that under the same conditions is just noticeably greater than r' and r_0' the analogue r_0. We shall define a particular case of zero difference of intensity by

Post. 10.— $$I_r - I_r' = 0.$$

We may now solve this equation for r. The solution is particularly simple when $r_0' = r_0$, and as these may be anticipated to be very close-lying quantities, we might for practical purposes identify them with their mean $\bar{r}_0$.

Writing for brevity

$$\frac{r_+ - \bar{r}_0}{r - \bar{r}_0} = k$$

we readily obtain

$$I_r = \left(\log \frac{k}{k'}\right)^{-1} \cdot \log \frac{r - \bar{r}_0}{r' - \bar{r}_0}.$$

The empirical question of the truth of Weber's law depends upon our ability to find conditions under which the subjective parameters k, r_0, r_ω remain constant. This is only a special case of the more general experimental problem, How do these parameters vary with the conditions of the experiment in which r is made to vary?

EDGAR A. SINGER, JR.

UNIVERSITY OF PENNSYLVANIA.

THE ISSUES CONCERNING MATERIAL IMPLICATION

IN a recent article in this JOURNAL,[1] Mr. Norbert Wiener objects to criticisms which I have made of the relation of material implication as the basis of inference. Mr. Wiener's paper fails to join the significant issue, but this may be as much my fault as his. If others have understood my papers as he seems to have done, then in justice to myself, as well as to those who are interested in Mr. Russell's logic, I should attempt some restatement. And the spirit of Mr. Wiener's criticism is so eminently just and fair that I take pleasure in pointing out my agreement with many of his contentions. For the sake of brevity and clearness, the matters to be considered may be summarized in set theses.

1. The relation of material implication, $p \supset q$, which figures in the mathematical logic in *Principia Mathematica*, is not the relation which we ordinarily have in mind when we say that q can be inferred from p.

[1] Vol. XIII., p. 656.